Math Out of the Box®: A K-5 Mathematics Curriculum and a Teacher Profession Development Program

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Background Information

Math Out of the Box is K-5 Mathematics curriculum and a companion professional development program for teachers under development in the College of Engineering and Science at Clemson University. The program's overarching goal is to fulfill the mathematical promise that exists in every child by providing teachers with innovative materials, a mathematically challenging curriculum, and high quality professional development.

Math Out of the Box is considered to be standards-based, research-based, and inquiry-based. A standards-based curriculum can be recognized by the inclusion of mathematics for all students, the inter-connectedness of processes and concepts, the connection to big ideas of mathematics, the continuous building of foundational ideas vertically through the grades, and the thoughtful identification of representations that build "intellectual engagement" (Trafton, Reys, and Wasman, 2001). Research-based curricula are those that are based on the body of knowledge that defines how students learn and how teachers teach. In addition, research-based curricula add to this body of knowledge in a continuous cycle of research and revision. Math Out of the Box is an inquiry-based curriculum in that it is designed so that students construct their own knowledge under the guided instruction of a teacher who has experienced similar knowledge construction.

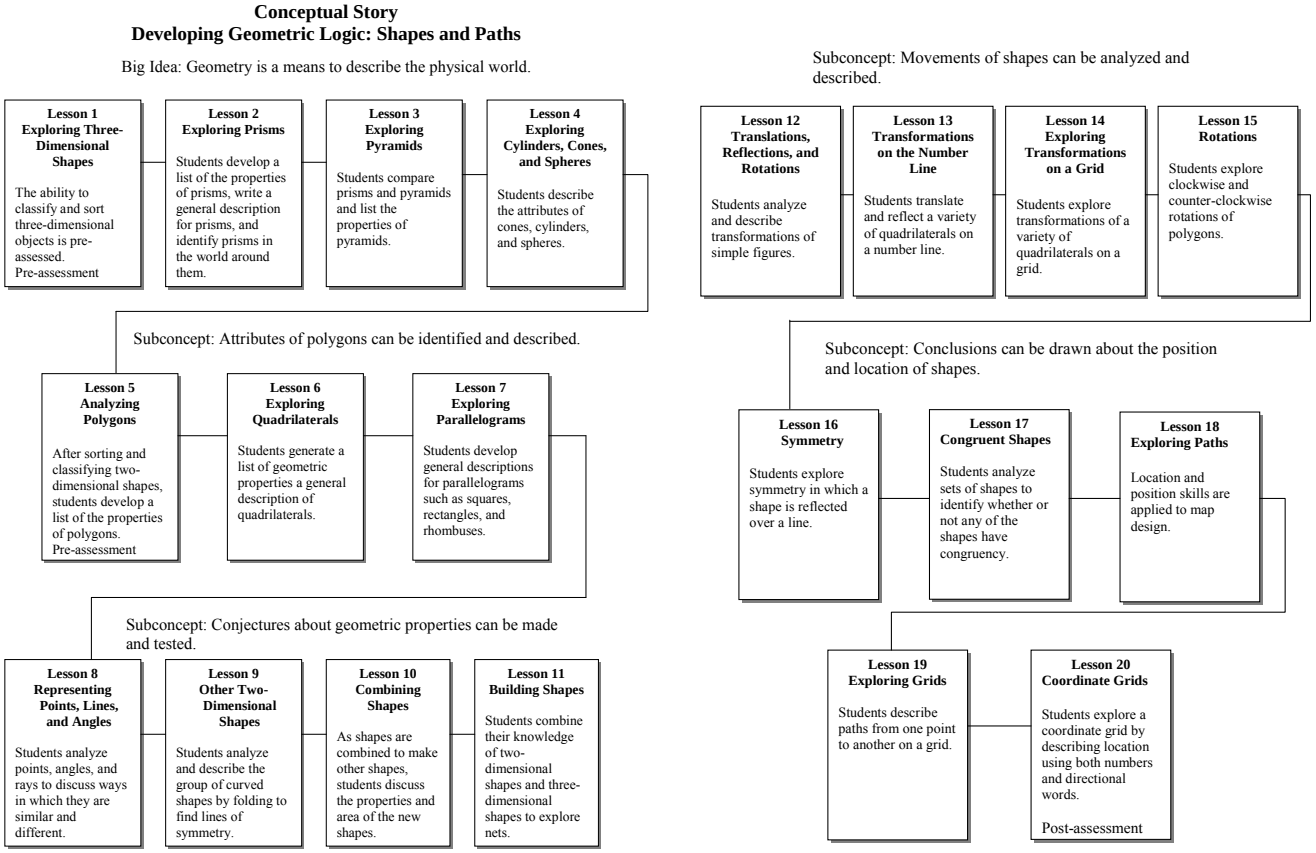
The Math Out of the Box developers have worked at all levels of K-16 education. As active participants in the decade-long South Carolina Statewide Systemic Initiative, the developers have extensive experience in designing and delivering professional development for education leaders, community organizations, and families. These professional development experiences include in-service and pre-service classes and workshops for mathematics and science teachers. As a result of these experiences in mathematics and science reform, the developers of Math Out of the Box formed the following beliefs about mathematics teaching and learning which are supported by the research-base of the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (NCTM, 2000):

- All students must have access to a curriculum that connects mathematical ideas.
- All teachers of mathematics need to be confident in their own teaching and learning as well as that of their students.
- Students need to have rich and varied experiences and materials as part of their mathematical learning.
- Assessment guides students in knowing what they have learned, aids teachers in planning instruction, and informs the community.

- Technology supports students and teachers as they engage in rich mathematical experiences.

The Math Out of the Box curriculum is designed to be released in four strands. The first two strands, *Algebraic Thinking* (which addresses algebra and data analysis standards) and *Geometric Logic* have been field tested and are currently available through the publisher Carolina Biological Supply Company. The last two strands, *Measurement Benchmarks* and *Number Concepts* are under development. The *Measurement Benchmarks* strand will be published early in 2007. The *Number Concepts* strand will be published beginning in 2008. These strands are vertically aligned through the grade levels and provide a comprehensive mathematics curriculum that is designed to support the mathematical development of all students, the professional development of teachers, and the development of the larger school community.

Each grade level manual is designed around a mathematical big idea and arranged in subconcepts. The following conceptual storyline shows the organizational design of third the grade geometry manual:



Math Out of the Box is developed through a rigorous process of research, development, lesson testing, and revision. The body of research on teaching and learning reviewed in the initial phase of the project included publications such as the following: *How People*

Learn: Brain, Mind, Experience, and School edited by John D. Bransford, Ann L. Brown, and Rodney R. Cocking: *Eager to Learn: Educating Our Preschoolers* edited by Barbara T. Bowman, M. Suzanne Donovan, and M. Susan Burns; and *Understanding by Design* by Grant Wiggins and Jay McTighe. Other sources which focused on the teaching and learning of mathematics included in the body of preliminary research were *Adding It Up: Helping Children Learn Mathematics* edited by Jeremy Kilpatrick, Jane Swafford, and Bradford Findell and the *Handbook of International Research in Mathematics Education* edited by Lyn D. English.

Standards and goals documents were reviewed as part of the research phase. The documents included the following:

- Project 2061, American Association for the Advancement of Science. *Benchmarks for Science Literacy.* New York: Oxford University Press. 1993.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics.* 2000.
- National Research Council. *Inquiry and the National Science Education Standards: A Guide for Teaching and Learning.* 2000.
- National Research Council. *National Science Education Standards.* 1996.
- International Society for Technology in Education. *National Educational Technology Standards.* 2000.

Learning Cycle

Math Out of the Box uses a learning cycle to foster inquiry-based learning. The learning cycle used in the lessons gives teachers the structure that is needed to reach their students, whether the teachers are traditional or inquiry-based in practice, providing teachers with a template that promotes the development of active inquiry and critical thinking. The learning cycle allows students to make connections between past and present learning experiences and is based in the "cognitive principle of assimilation," which implies that understanding cannot be imposed on the learner, but instead is developed progressively by the learner, beginning with concrete and progressing to abstract opportunities. The learning cycle provides the opportunity for students to share ideas with others and to more formerly connect what they have learned with what they already know.

The engage phase of the learning cycle allows students with varying prior experiences and abilities to make connections between past and present learning experiences. Such connections provide a natural pre-assessment opportunity for both the teacher and student.

In the investigate phase of the learning cycle students are given common, concrete experiences that challenge them to solve problems and investigate mathematical ideas. Information is gathered, patterns are observed and analyzed, connections are made and applied, and conclusions are drawn and defended.

The reflect phase of the learning cycle is where students think about how what they have learned fits into what they already know. The teachers' role is especially crucial because it is their knowledge of mathematics that enables them to assist students in summarizing and structuring their thinking into meaningful models of the mathematical ideas they have explored.

In the application phase of the cycle, students and teachers can assess the depth of understanding of the newly formed ideas. The new knowledge then becomes old knowledge on which to connect new learning—and the cycle of learning continues.

In addition to the learning cycle, the Math Out of the Box curriculum is designed around several components essential to inquiry. These components include:

Development of a community of learners

Extensive research corroborates the effectiveness of collaborative groups in K-5 classrooms and their use to build a learning community. After examining the large body of research on cooperative groups, one group of researchers conclude that "Markedly different theoretical perspectives (social interdependence, cognitive-developmental, and behavioral learning) provide a clear rationale as to why cooperative efforts are essential for maximizing learning and ensuring healthy cognitive and social development as well as many other instructional outcomes" (Johnson, Johnson, and Stanne, 2000, p.9).

A model for verbal and written communication

The communication model in Math Out of the Box lessons provides a structure for successful verbal and written experiences throughout each mathematical subconcept. Discussion, questioning, reflection, and writing are communication strategies that ensure that meaningful mathematical thinking occurs in mathematics classrooms. Communication in the mathematics classroom permits learning to build on the students' informal knowledge, gives students practice in explaining their mathematical thinking to others, and provides students and teachers with evidence that learning has occurred. (Yackel , Cobb, Wood, and Merkel, 1990; Malloy, 1997).

Explicit connections that make mathematics meaningful

The Math Out of the Box curriculum is designed so that students will develop the ability to make meaningful mathematical connections. The ability to recognize relationships among mathematical ideas and to apply those ideas beyond the mathematics classroom has long been recognized as a hallmark of mathematical understanding (Brownell, 1954; Skemp, 1978; Grouws & Cebulla; 2000). In recent years, the ability to recognize such relationships is often referred to as "making mathematical connections." The benefits of mathematical connections in developing mathematical understanding is well documented in cognitive psychology, and is recognized as an essential part of learning mathematics by mathematics teachers and educators (Stigler and Hiebert, 2004). In the 2004 TIMSS Video Study, the making of connections among mathematical ideas was cited as the most

significant feature distinguishing the higher-achieving countries' mathematics instruction from the other countries in the study (Stigler and Hiebert, 2004). The importance of making mathematical connections in developing mathematical fluency cannot be overstated.

Balanced assessment practices

Assessment is an ongoing, essential component of the inquiry-based learning cycle used in the Math Out of the Box curriculum. Assessments are built around concepts and skills based on mathematical standards. The goals for assessment in the Math Out of the Box curriculum are

- to guide students in knowing what they have learned.
- to allow the teacher to understand how students are thinking about mathematics.
- to aid teachers in planning instruction.
- to inform the community.

Two types of assessment are used throughout the lessons. Formative assessments are embedded into the lessons, providing information to the teacher for instructional decisions and information to the students about their own learning. Numerous studies support the practice of formative assessment as a way to increase student success, particularly with low-achieving students (Fuchs and Fuchs, 1986; Wiliam and Black, 1996). Summative assessments provide additional information about student learning and can be evaluative in nature, providing information to a broader community. A variety of assessment strategies are included in each lesson to allow students multiple opportunities to demonstrate their knowledge and skills. Lessons that function as performance assessments are included in each module.

A variety of problem solving experiences

Data from reform curricula of the 1990s indicate that students using curricula, with an emphasis on problem solving, perform as well as students using traditional curricula on basic skills and better on conceptual understanding on standardized tests (Schoenfeld, 2002; Senk and Thompson, 2003). Research indicates that opportunities to explore new ideas balanced with opportunities to practice skills results in successful problem solving (Grouws and Cebulla, 2000).

Researchers continue to find a relationship between the development of students as thinkers and student success in problem solving and conceptual understanding. Studies have examined the issues in classroom application when problem solving is considered as a process rather than another topic in a mathematics curriculum (Fennema, Carpenter, and Lamon, 1996; Kazemi, 1998; Kennedy, Tipps, and Johnson, 2004). The following beliefs of the developers of the Math Out of the Box curriculum are based on such research:

- A safe environment must be developed as part of the learning community so that mathematical discourse can take place.
- Changes in thinking can occur as errors and misconceptions are reconceptualized.
- Successful problem solving often requires multiple attempts and multiple strategies.
- Problem solving as a community leads to shared understanding of mathematical ideas, individual accountability, and connections to life outside of the mathematics classroom.

A diversity of materials, manipulatives, and models

Researchers advocate an environment of hands-on experiences in mathematics classrooms. In addition to manipulatives, materials needed for this rich environment include charts, graphs, writing models, diagrams, technology, and any tool that aids students in sense-making and problem solving (Sowell, 1989; Hiebert et al., 1997; Kilpatrick, Swafford, and Findell, 2001; Van de Walle, 2004).

Each Math Out of the Box unit includes a teacher's manual with student blackline masters and a kit of materials needed to effectively teach the lessons. Including the materials as part of the curriculum and in professional development, ensures that materials are used effectively by students to demonstrate and develop knowledge, to self-assess learning, and to connect mathematic ideas. Embedding the use of materials throughout the learning cycle of each lesson provides a powerful means of formative assessment for the teacher as students investigate mathematical ideas.

Information About Professional Development

Innovative and meaningful professional development experiences provided in partnership with established organizational structures are needed for successful implementation of any curriculum (DuFour and Baker, 1998; Loucks-Horsley, Hewson, Love, and Stiles,1998; Guskey, 2000). As strands of the Math Out of the Box curriculum are developed, correlating professional development experiences in partnership with the publisher and other organizations are also developed, field-tested, and revised.

In addition to these professional development experiences, the Math Out of the Box curriculum includes embedded strategies to support and change teachers' knowledge and beliefs about mathematics. Throughout the lessons, procedures and processes of effective teaching are modeled for teachers including effective questioning, writing strategies, representation as a key to successful problem-solving, and reflective practices. The learning cycle that is used to organize the lessons in the Math Out of the Box curriculum provides teachers with an effective mechanism for including inquiry-based practices, such as formative and summative assessments, throughout the mathematics lessons. Research shows that teachers' knowledge and belief systems can be affected by such

experiences (Fullan, 1982; Cohen and Ball, 1990; Fennema, Carpenter, and Lamon, 1991).

A variety of professional development models are offered to school districts participating in research and implementation projects. An emphasis on mathematical content and the necessary pedagogical strategies for implementing an inquiry-based curriculum are provided to teachers. The mathematical concepts are developed vertically so that teachers recognize their role in building foundations and making conceptual connections. The professional development immerses teachers, working together in their grade level teams, in the standards and mathematics of the strand, provides a model for implementation of research-based strategies, and provides a vertical overview of the strand. Teachers are introduced to a structure for team planning and reflection.

The current professional development program supports initial implementation of Math Out of the Box. Plans to expand this program to include parents, business leaders, and other community members in supporting the implementation process are in the works. Additionally, there are plans to create a program of professional development for sustainability. This program will support schools and districts beyond the initial implementation phase and will focus on a number of areas including deepening the mathematical content knowledge of elementary teachers, developing meaningful and informative assessment practices, and improving instructional practices so that all students are engaged in the process of learning.

Program Assessment-Filed Tests

Data collected from field test projects inform the final revision of the curriculum before publication. Decisions about inclusion and quality of materials are made based on feedback from field test teachers. Changes in instructional lessons are made based on teacher feedback and data collected during classroom observations. Problematic issues may be addressed directly in the curriculum through revisions, or they be addressed through the professional development program, or both. For example, teachers report that materials management is an issue at first. Using suggestions from teachers, the developers include helpful information in the lessons. During professional development, materials management is addressed and teachers are provided with an opportunity to plan together to prepare for managing the materials efficiently to meet their personal needs.

Reflections from field test teachers are collected electronically. Following are several examples from the third grade field test for Developing Algebraic Thinking: Plotting and Growing:

> Dear Teachers,
> We hope you have had great experiences with the first two lessons in the third grade manual. Please respond electronically to the following prompts:
>
> Please share something that you learned about your students that had to do

with the math in the lessons. Also share something you learned about your students as a result of the kit lessons that wasn't mathematical.

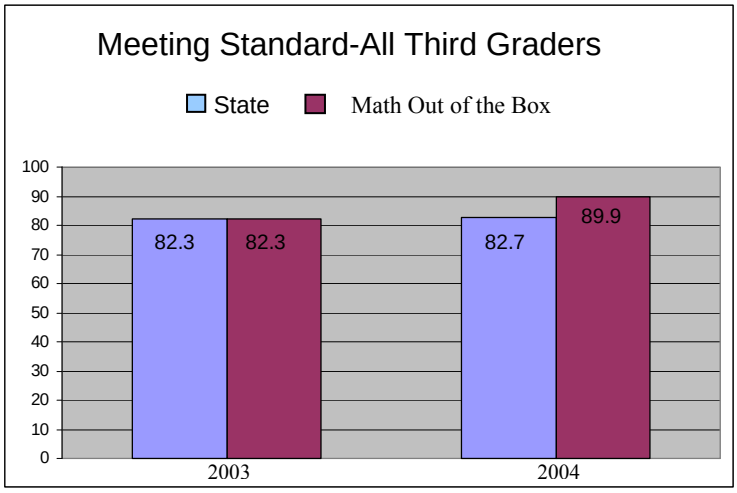
From K. Senger
I learned that my kids had no idea how to use a tape measure. We have not studied measurement yet and several didn't even know which end to start with. The number 1 was not enough of a hint! My class is also very talkative and I thought the group work would be a challenge for them. So far it has gone okay and since the groups only have 2 in them they seem to work better than in larger groups. I have enjoyed the kit so far and I think the kids have too.

From N. Wolfe
I learned that my students knew how to measure straight objects using inches and centimeters, but that it was difficult for some of the students to figure out how to measure something round. The students enjoyed working with a partner very much this week. All students had fun in math. For the first time this year my lower level students could participate equally with the faster achieving students. I learned that each of my students would participate in a group if they felt comfortable with the assignment.

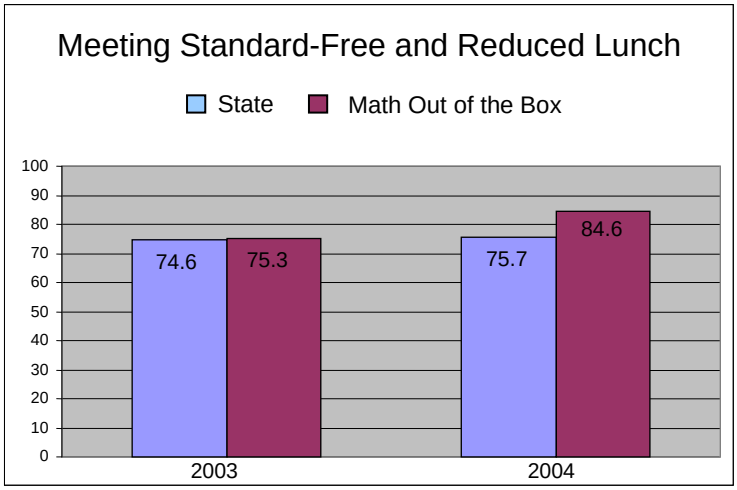
From M. Jackson
My kids have loved using the kit so far. I was very surprised that my kids already knew a lot about data. I even had one who gave me the same definition that is listed in the kit. They were also very aware of how important it is to test things in a fair manner. After planting the bulbs, they were full of even more ideas on why it is so important to have fair tests. I had one child tell me that it was a good thing that everyone was here today to plant their bulbs, because if someone had been absent, it wouldn't have been fair. I asked her to explain why, and she did it correctly even using the word data in her explanation. In addition, I have learned that I have numerous kids in my class that want to be the "leader." I guess I have always known that, but during these first lessons, it has become more evident. That makes it really hard to work in groups. That is one thing I have been struggling with. I am still trying to match up partners and groups. We are getting there though. Overall, I am pleased with the kit. The kids are having fun and learning at the same time.

Student achievement data is collected from a variety of sources during field tests. Pre- and post-assessment items are included in each teacher's manual, many schools and districts have a local diagnostic test, and the No Child Left Behind Act requires extensive testing. Following are third grade field test scores for Developing Algebraic Thinking: Plotting and Growing from South Carolina's test, the Palmetto Achievement Challenge Test (PACT)

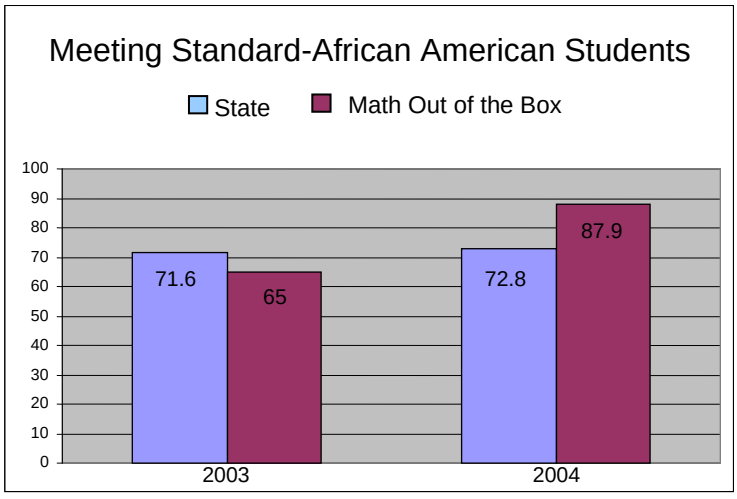
Third Grade Field Test Results



89.9% of the 250 Math Out of the Box third graders from the three field test schools met standard on the mathematics section of the third grade PACT in 2004 compared to the state rate of 82.7%.



50% of the third graders in the Math Out of the Box classrooms were eligible for the free and reduced lunch program. Of those students, 84.6% met the standard on the mathematics section of the third grade PACT compared to the state rate of 75.7%.



40% of the third graders in the Math Out of the Box classrooms were African American students. Of those students, 87.9% met the standard on the mathematics portion of the third grade PACT compared to the state rate of 72.8%.

Program Assessment –Pilot Projects

The pilot research projects are designed to provide data that enables developers to make formative assessments concerning content issues, instructional issues, materials management issues, and implementation issues. This information primarily informs the content of the professional development program. Some issues are unique to schools and addressed as such. Other issues are more universal, such as the lack of content knowledge of teachers. A plan to develop a more comprehensive, content-focused program of professional development is currently in the works. This program of professional development will be made available to schools and districts who have adopted Math Out of the Box to support them in continuing to improve teaching effectiveness and student achievement outcomes.

Teachers use a pre/post retrospective to make comparisons about their role and the role of their students before and after implementation. Student work brought to this session is displayed and analyzed showing the conceptual story of the kit and the vertical development of mathematics concepts. Teachers explore a middle school problem and reflect on their role in preparing students to be successful solvers of such problems.

Assessment data for teachers has been collected predominantly from pilot sites and has focused on the teachers' instructional practices. During full group interviews that are conducted in the early weeks of implementation, most teachers voice frustration with the inquiry-based expectations of the curriculum. Data collected from observations of classrooms during the early weeks of implementation provides evidence that the curriculum supports most teachers in creating inquiry-based environments; however, the weak mathematical content knowledge of the teacher often interferes with the teacher's ability to utilize student contributions in meaningful and instructive ways. After teachers have implemented a complete strand (in most cases the Algebra/Data strand), they report that they have learned "a lot of math" and that they have learned more about their students than they ever knew before. Observations conducted toward the end of the unit indicate that teachers have become more adept at supporting inquiry-based environments, and they appear to be more comfortable with their own learning in that environment. Teachers who plan regularly with grade level peers appear to have the most positive experiences in terms of their own learning and in terms of improvement in instructional and assessment practices.

Following are examples of written reflections collected from teachers involved in pilot implementation projects:

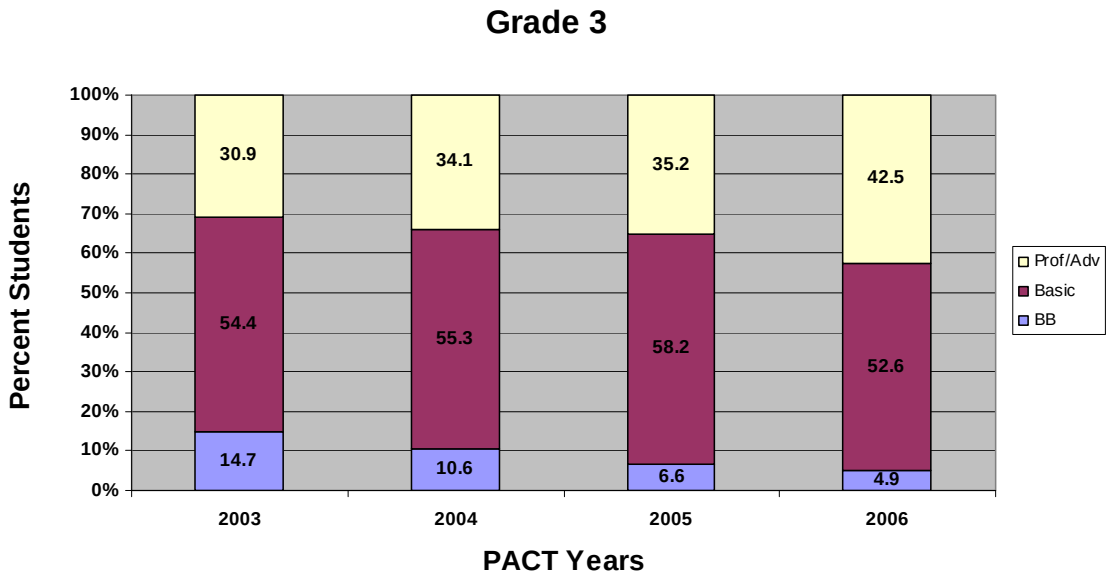
4th grade teacher
"I was excited about the depth of understanding that I gained in mathematics from the professional development and implementation of the Math Out of the Box program. I became more confident at teaching a variety of mathematical concepts such as algebra and data…I became more a risk-taker [and] allowed my students more exploration time and freedom to arrive at answers in multiple ways. I was astounded at the students'

creativity throughout the implementation of Math Out of the Box. The most rewarding experience BY FAR was the overwhelming enthusiasm students had for math with this project. Lower level students were given frequent opportunities to experience success, which helped them to really take ownership of their learning."

4th Grade Teacher

My students have shown me, through MOOTB that the "right" answer to a problem can come in many forms. It's interesting to experience how people can see things differently and still be right. The kids teach me and show me new ways of seeing new concepts. Sometimes they make an observation before I even begin the formal lesson. They know I'm learning too−so they are patient with me. Thank Goodness!

Change takes time. The following student test data collected over time at a pilot site where each strand of MOOTB was implemented along with the companion professional development indicates that as a change in teacher content knowledge and pedagogy takes place, the goal of student proficiency in mathematics is more likely to be met. In 2004, the teachers at this school taught one strand of Math Out of the Box, followed by two strands in 2005, and three strands in 2006. The top part of each column shows the students who scored proficient and advanced, the middle part shows students who scored basic, and the bottom part, students who scored below basic on the Palmetto Achievement Challenge Test.



Math Out of the Box Partners

Math Out of the Box is considered a pre-engineering elementary mathematics curriculum. The business and industry partners who provide funding for the project have an interest in a diverse workforce and recognize that elementary school mathematics is an important piece of the education pipeline. Corporations and foundations including Dupont Office of

Education, Michelin North America, Fluor Daniel, American Honda Foundation, John Deere Foundation, Self Family Foundation, Ford Motor Foundation, General Electric Fund provide financing for pilot programs and research projects.

Educational Testing Service creates assessment items, tests evaluation instruments, collects and analyzes qualitative and quantitative data, and evaluates the Lawrence Township, NJ project.

Carolina Biological Supply Company publishes the curriculum, provides financial support to Clemson University, develops the hands-on materials in partnership with Clemson University, and provides in-kind support for field tests and pilots.

Math Out of the Box field test and pilot programs have taken place in South Carolina, Pennsylvania, New Jersey, and Illinois in over 500 classrooms with over 11,000 students.

Bibliography

Black, P. and D. Wiliam. (1998). Assessment and Classroom Learning. *Educational Assessment: Principles, Policy, and Practice*. 5 (1): 7-74.

Bowman, B.T., M.S. Donovan, and M.S. Burns, eds. (2000). *Eager to Learn: Educating Our Preschoolers.* Washington, DC: National Academy Press.

Bransford, J.D., A.L. Brown, and R. Cocking, eds. (2000). *How People Learn: Brain, Mind, Experience, and School*. Washington, DC: National Academy Press.

Brownell, W. (1987, April, originally 1956, October). Meaning and Skill: Maintaining the Balance. *The Arithmetic Teacher*, 18-25.

Cohen, D. and Ball, D. (1990). Relationships Between Policy and Practice: A Commentary. *Educational Evaluation and Policy Analysis*. 12(3): 331-337.

Devlin, K. J. (2002). *The Language of Mathematics: Making the Invisible Visible*. New York: Henry Holt and Company.

Diaz, D.P. (2004). *The Role of Standards-Based Curriculum in Teacher Development and Instructional Reform*. Dissertation: Clemson University.

DuFour, R.and R. Eaker. (1998). *Professional Learning Communities at Work: Best Practices for Enhancing Student Achievement.* Bloomington, IN: National Education Service and Alexandria, VA: Association for Supervision and Curriculum Development.

English, L.D., ed. (2002). *Handbook of International Research in Mathematics Education*. Reston, VA: National Council of Teachers of Mathematics and Mahweh, NJ: Lawrence Erlbaum Associates.

Fennema, E., T.P. Carpenter, and S.J.Lamon, eds. (1991). *Integrating Research on Teaching and Learning Mathematics*. New York: State University of New York Press.

Fullan, M.G. (1982). *The Meaning of Educational Change.* New York: Teachers College Press.

Fuchs, L.S. and D. Fuchs. (1986). Effects of Systematic Formative Evaluation: A Meta-Analysis. *Exceptional Children*. 53: 199-208.

Grouws, D. and Cebulla, K. (2000). *Improving Student Achievement in Mathematics*. Brussels: International Academy of Education. Retrieved February 14, 2005, from http://www.ibe.unesco.org/International/Publications/EducationalPractices/EducationalPracticesSeriesPdf/prac04e.pdf.

Guskey, T.R. (2000). *Evaluating Professional Development*. Thousand Oaks, CA: Corwin Press.

Hiebert, J., T.P. Carpenter, E. Fennema, K.C. Fuson, D. Wearne, H. Murray, A. Olivier, and P. Human. (1997). *Making Sense: Teaching and Learning Mathematics with Understanding.* Heinemann: Portsmouth, NH.

International Society for Technology in Education (ISTE). (2000). *National Educational Technology Standards for Students-Connecting Curriculum and Technology*. Eugene, OR: International Society for Technology in Education.

Jarrett, Denise. (1997) Inquiry Strategies for Science and Mathematics Learning: it's Just Good teaching. Portland, OR: Northwest Regional Educational Laboratory.

Johnson, D.W., R.T. Johnson, and M.B. Stanne. (2000). *Cooperative Learning Methods: A Meta-Analysis.* Minneapolis, MN: University of Minnesota.

Karplus, R., and H. D. Their. (1967). *A New Look at Elementary School Science*. Chicago, IL: Rand McNally.

Kazemi, E. (2002). Discourse That Promotes Conceptual Understanding. In Chambers, D. L., ed. *Putting Research into Practice in the Elementary Grades: Readings from Journals of the National Council of Teachers of Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Kennedy, L., S. Tipps, and A. Johnson. (2004). *Guiding Children's Learning of Mathematics*. Belmont, CA: Thomson/Wadsworth Learning.

Kilpatrick, J., J. Swafford, and B. Findell, eds. (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Lawson, A. E., M.R. Abraham, and J. W. Renner. (1989). A Theory of Instruction: Using the Learning Cycle to Teach Science Concepts and Thinking Skills. National Association of Research in Science Teaching (NARST) Monograph, Number One.

Loucks-Horsley, S., P. W. Hewson, N. Love, and K. E. Stiles. (1998). *Designing Professional Development for Teachers of Science and Mathematics*. The National Institute for Science Education. Thousand Oaks, CA: Corwin Press, Inc.

Lowery, L.F. (1989, reprinted 1998). *Thinking and Learning: Matching Developmental Stages with Curriuclum and Instruction.* Seattle, WA: Teachers Press.

Malloy, C.E., ed. (1997). *Challenges in the Mathematics Education of African American Children.* Proceedings of the Benjamin Banneker Association Leadership Conference. Reston, VA: National Council of Teachers of Mathematics.

Marek, E.A., and A. M.L. Cavello. (1997). *The Learning Cycle: Elementary School Science and Beyond*. Portsmouth, NH: Heinemann.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (2000). *Inquiry and the National Science Education Standards: A Guide for Teaching and Learning.* Washington, DC: National Academy Press.

National Research Council. (1996). *National Science Education Standards.* Washington, DC: National Academy Press.

Pellegrino, J. W., N. Chudowsky, and R. Glaser, eds. (2001). *Knowing What Students Know: The Science and Design of Educational Assessment*. Washington, DC: National Academy Press.

Project 2061, American Association for the Advancement of Science. (1993). *Benchmarks for Science Literacy*. New York: Oxford University Press.

Schmidt, W., R. Houang, and L. Cogan. (2002). *A Coherent Curriculum: The Case of Mathematics*. American Educator. Summer.

Schoenfeld, A. H., (2002). Making Mathematics Work for All Children: Issues of Standards, Testing, and Equity. *Educational Researcher*. 31. Number 1, pp. 13-25.

Senk, S. L. and D. R. Thompson, eds. (2003). *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* Mahweh, NJ: Lawrence Erlbaum Associates.

Sowell, E.J. (1989). Effects of Manipulative Materials in Mathematics Instruction. *Journal for Research in Mathematics Education.* 20: 498-505.

Stigler, J.W. & Hiebert, J. (2004). Improving Mathematics Teaching. *Educational Leadership.* 61(5), 12-17.

Trafton, Paul R., Barbara J. Reys, and Deanna Wasman. (2001) Standards-Based Mathematics Curriculum Materials: A Phrase in Search of a Definition. *Phi Delta Kappan*, 83: 259-264.

Van de Walle, J. A. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally,* Fifth Edition. Boston, MA: Pearson Education, Inc.

Wiggins, G., and J. McTighe. (2001). *Understanding by Design*. Upper Saddle River, NJ: Merrill/Prentice Hall.

Wiliam, D. and P. Black. (1996). Meanings and Consequences: A Basis for Distinguishing Formative and Summative Functions of Assessment. *British Educational Research Journal*. 22: 537-48.

Yackel, E., P. Cobb, T. Wood, and G. Merkel. (2002). Experience, Problem Solving, and Discourse as Central Aspects of Constructivism. In Chambers, D. L., ed. *Putting Research into Practice in the Elementary Grades: Readings from Journals of the National Council of Teachers of Mathematics*. National Council of Teachers of Mathematics. Reston, VA.