Impacting Instructional Practice through the Implementation of an Inquiry-Based Elementary Mathematics Program: A Single Site Collective Case Study

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abstract:

Traditional K-12 mathematics instruction begins with rules and procedures and progresses to applications of those rules and procedures. Inquiry based instructional practices engage students in the discovery of rules and procedures through mathematical investigations. Most elementary teachers practice traditional mathematics instruction although many might describe themselves as inquiry-based practitioners. This study examines how the implementation of an inquiry-based mathematics curriculum impacts the instructional practices of K-5 educators in a Title I school district. The purpose of this paper is to describe the changes in practice that occurred throughout the implementation process and to outline several strategies that aided teachers while making the transition from traditional to inquiry-based practitioners.

introduction:

According to the Building Engineering and Science Talent (BEST) report, "Twenty-five percent of our scientist and engineers will reach retirement age by 2010" (p. 1)[1]. The prevailing concern that American students are not as prepared to meet the challenge of scientific innovation when compared to students in other nations has prompted a response from the federal government. An abundance of federal funding has been allocated towards preparing our students, teachers, and future professionals in the areas of science, technology, engineering, and mathematics (STEM)[2]. Much of the research associated with this funding has focused on K-12 education and more specifically with increasing student achievement in STEM areas beginning in Kindergarten. This growing concern to increase student achievement has resulted in a push for practitioners to utilize what has been termed, inquiry instruction, in the classroom.

The term inquiry has been used in numerous journal articles, textbooks, pre-service education courses, and professional development workshops. The National Research Council identified the use of inquiry as an integral part of increasing student achievement when creating the National Science Education Standards. "Inquiry is essential to learning. When engaging in inquiry, students describe objects and events, ask questions, construct explanations, test those explanations against current scientific knowledge, and communicate their ideas to others" (p.2)[3]. Inquiry has commonly been defined as based on the theory of constructivism where students develop knowledge through experiences or learn by doing[4,5 6, 7]. The purpose of this paper is to examine what traditional and inquiry practice look like in a classroom setting in which an inquiry-based mathematics curriculum is first being introduced. The intention here is not to prove that one approach is better than the other in terms of student achievement or motivation, it is to examine changes in instruction when teachers implement an inquiry-based program.

To examine what traditional and inquiry practice look like in a classroom, it is necessary to first define these terms. As stated previously, inquiry is most commonly associated with the theory of constructivism. Teaching through inquiry has its roots in education as early as the beginning of the nineteenth century with John Dewey and his laboratory school[8]. Theorists like Piaget, Vygotsky, and Bruner examined cognitive development and advocated an active educational setting where students construct their own knowledge with guidance from their teachers[9]. Battista describes traditional mathematics instruction as being "almost identical to what most adults were taught when they were children"(p.428)[10]. In traditional instruction, students are presented with a rule, such as an algorithm that they can apply to examples provided during individual practice. Students are expected to replicate procedures and facts that were introduced by their teacher[11]. This practice differs from inquiry instruction where students are presented with situations where they must discover a rule through mathematical investigations.

There have been ongoing debates about what types of practice are appropriate in mathematics instruction[12,13]. Again, the intention here is not to describe one practice as being superior, but rather to examine how teachers in this setting changed their instructional techniques from traditional teaching to inquiry-based instruction when implementing the Math Out of the Box® program, an inquiry-based mathematics curriculum under development through Clemson University. These styles of instruction seem to be in opposition to each other, one being centered on the development of knowledge and one with the transmission of knowledge. It is surprising to note how through many observations during the course of this study, teachers were identified as practicing both traditional and inquiry methods within a single lesson. This phenomenon indicates that it may be difficult for current teachers to make the transition from traditional instruction to the inquiry-based instruction that many school districts are advocating.

program implementation:

Math Out of the Box is a K-5 mathematics curriculum designed with five interrelated strands. The first published strand, Developing Algebraic Thinking with Patterns and Data, is broken down into twenty lessons at each grade level with each lesson requiring at least an hour to complete. The first ten lessons of this strand focus on Algebra concepts and the second ten on concepts dealing with Data. The Developing Algebraic Thinking with Patterns and Data strand was implemented during the course of this study. Teachers were trained on the Math Out of the Box materials through an inquiry-based professional development model where they were immersed in tasks in which the facilitator supported an inquiry-based learning environment.

The professional development model consisted of two full days of inquiry experience and a half-day at the end of implementation dedicated to reflection of practice. The first day of professional development focused mainly on Algebra concepts and was given prior to implementing any of the Math Out of the Box lessons. After teachers implemented the ten lessons relating to Algebra, they returned for the second day of professional development dealing primarily with data concepts. Teachers were also given the opportunity to reflect on the Algebra lessons and discuss issues relating to implementation with their peers. Topics such as materials management, timing of lessons, and mathematical content were commonly discussed among grade groups during the second workshop. After the participants taught the ten lessons relating to Data, they returned for a third session, which provided an opportunity for reflection on the whole implementation process. Teachers were asked to share their thoughts and experiences with each other and complete a questionnaire in which they compared classroom instruction before Math Out of the Box and instruction during Math Out of the Box. Teachers commonly recognized a deepening of their own mathematical content knowledge through participation in this program. “I learned a lot about how to teach math in a more understandable, hands-on way. Math was my weakest subject as a child and this program opened my eyes to how involved and easy math can be. I learned a lot about patterns. The terms were words I had never associated with math.” Throughout this whole process, the facilitators of the workshops acted as guides, allowing teachers to construct their own meaning about what was required to implement the program effectively and providing questions to facilitate discussion. One teacher commented on this reflection process on her questionnaire by writing, “I learned how helpful it is to get feedback from others. It helps to communicate ideas that work and fail”.

methodology:

This paper describes a single-site collective case study conducted in the second, third, and fourth grade classrooms of a public elementary school in South Carolina. Schram states that a case study should involve, “the exploration of a ‘bounded system’, something identifiably set within time and circumstance” (p. 107)[14]. This study examines the question: How do teachers change their instructional practice when implementing an inquiry-based mathematics curriculum and what supports are needed to implement this program effectively? Yin identifies “how” or “what” questions as being natural for a case study design[15].

The study took place in a single setting, but examined multiple cases. Nineteen teachers participated in the implementation process; seven teachers at the second grade level, six at the third grade level, and six at the fourth grade level. Education levels among the participants range from obtaining a Bachelor’s degree to a Master’s Degree and the level of experience varied from one year to over twenty years in the elementary classroom. Issues of reliability and validity were addressed in this study by using multiple cases rather than focusing on one particular teacher. By looking at more than one case, the researcher was able to draw conclusions that have been replicated within the study, indicating that the conclusions drawn during this study are stronger than if only one teacher was examined[15].

The specific setting where this research took place is a public elementary school in South Carolina. This elementary school is one of forty-nine schools in the district. According to the South Carolina Department of Education, there are currently five hundred and ninety students enrolled in the school from kindergarten to fifth grade. Currently, the school has an unsatisfactory rating on the statewide report card and has not met all of the objectives required by the state for Adequate Yearly Progress[16]. This school was awarded a grant to implement the Math Out of the Box program with the purpose of impacting instruction and student achievement in mathematics. The participating teachers were selected according to criterion sampling strategies; they had to be a teacher in this particular setting that was willing to implement the new math program in the classroom. It is important to note that the unit of analysis is not the school where this study is taking place[15]. Each individual teacher represents a specific case making up a multiple case study within a single setting.

One of the strengths associated with using a case study as a means of research is the variety of data that a researcher must collect to draw conclusions[14,15]. Most case studies require four to six forms of data to be considered a comprehensive qualitative study. The data collection methods for this study are as follows:

1. Observations of participants prior to implementing the new mathematics curriculum.
2. Observations of participants while implementing the new mathematics curriculum.
3. Focus group interview of fourth grade teachers during a collaborative planning session.
4. Written reactions to the program by participants after completing the new mathematics curriculum.

The use of multiple forms of data aided in maintaining credibility and validity during the course of this research. Each form of data may have strengths and weaknesses. By utilizing so many forms, these weaknesses can be accounted for within the study[15,17].

Due to the nature of the elementary school schedule, not all teachers could be observed before teaching Math Out of the Box. There were, however, enough pre-observations collected to identify themes among participants that typify instruction before implementation. Each teacher was observed at least one time and at the most three times over the course of two months in the spring of 2006. Upon completion of the program, teachers were asked to complete a questionnaire (found in Appendix A) and to participate in a focus group examining their planning practices prior to instruction.

Data analysis occurred in two separate steps. First, categories were developed by coding data from each observation to identify preliminary themes[15,18]. The categories that were identified describe different dimensions of instructional practice that can occur in the classroom. These dimensions include student and teacher roles in the classroom, the types of tasks that can occur during instruction, the use of manipulatives during lessons, teacher beliefs about mathematics, and content misconceptions that teachers may demonstrate during the lesson. After the categories were established, data was further analyzed to determine if the practice under each category was considered traditional instruction or inquiry instruction. An instructional analysis instrument, Five Dimensions of Instructional Practice (FDIP), designed by a member of the development team for the purpose of documenting fidelity of implementation of the curriculum was used for analyzing and describing instructional practice in each category. (Appendix B contains a copy of the instrument) The instrument was designed based on a synthesis of information from mathematics educators[19,20], researchers[21,22,23], and philosophers[24]. The instrument has provided an effective and consistent basis for categorizing instructional practices in other research settings. Educational Testing Service (ETS) adapted the instrument for use in an independent evaluation of the fidelity of implementation in Lawrence Township Public Schools, which further established the instrument's reliability in documenting instructional practice. After each observation was coded and the data was identified as either traditional or inquiry, researchers utilized the remaining forms of data to triangulate results and to identify means of support that teachers required when implementing an inquiry-based mathematics curriculum[18].

results:

Six themes emerged during the initial data analysis phase when coding the observations of participants prior to and during implementation of the Math Out of the Box program. These themes are outlined in Figure A with examples from the data. The examples found under

Teacher Misconceptions might be difficult to interpret as a lack of content knowledge on the part of the teacher. Nuances such as body language, pauses in conversation, and difficulty in responding to questions are hard to capture through records of observations. It could be beneficial in future studies to use a narrative form of research when conducting classroom observations.

Figure A

Themes Emerging from Qualitative Data

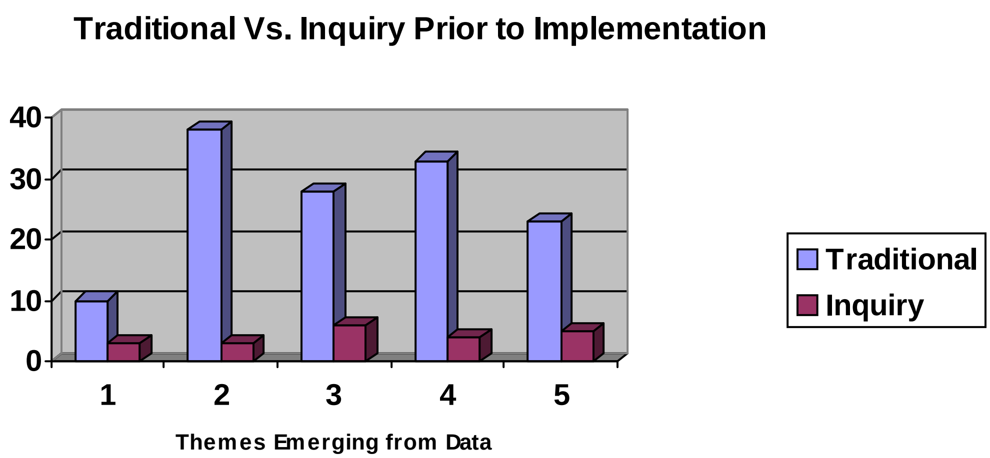
	Use of Manipu-latives	**Math as a Discipline**	**Student Tasks**	**Teacher Role**	**Student Role**	**Teacher Misconcep-tions**
Prior to Math Out of the Box	We don't have enough rods for everyone so we are not going to use them. 2.3A Write the number 34 on the whiteboard. 2.3A Let's look at the number 47. Put your finger on number 47 on the number chart. 2.6A Take your animals and put them into four even groups. Look at a group and tell me how many are in one group. 4.3 A	I don't want you to subtract, I only want you to show me how to regroup the number 34. 2.3A When we subtract, do we start in the tens column or the ones column? 2.6A Congruent means exactly the same size and shape. 3.2A Put the fractions in order from least to greatest. 3.3A	When I call your number, come up and get a review sheet and go to your test spot to work on it. 2.6A Work on numbers 1-14 on page 499. This is my way of seeing how you will do on your quiz tomorrow. 3.3A Write this fraction on your board and put it in the simplest form. 4.3A	We have to cut up one of our tens so it turns into ones. 2.3A Eyes on me and tell me what we did today in math. 2.6A Ordering fractions is putting them in order from greatest to least and least to greatest. 3.3A This is one whole square. If we divide it in half, we have two pieces. 4.6A	Put your thumbs up if you agree with him or down if you disagree with him. 2.6A Very good! She did a really good job of shading them in. All right, which one is the least amount? 3.3A Student writes the decimal .75. Another student corrects him stating it should be 0.75. 4.6A	Student puts fractions in order from least to greatest: 2/3, ¾, 5/8. Teacher changes it to 2/3, 5/8, ¾. 3.3A Teacher has students solve 42-17. Student does it on the board. She tells the student that you can't subtract 2-7. 2.3A

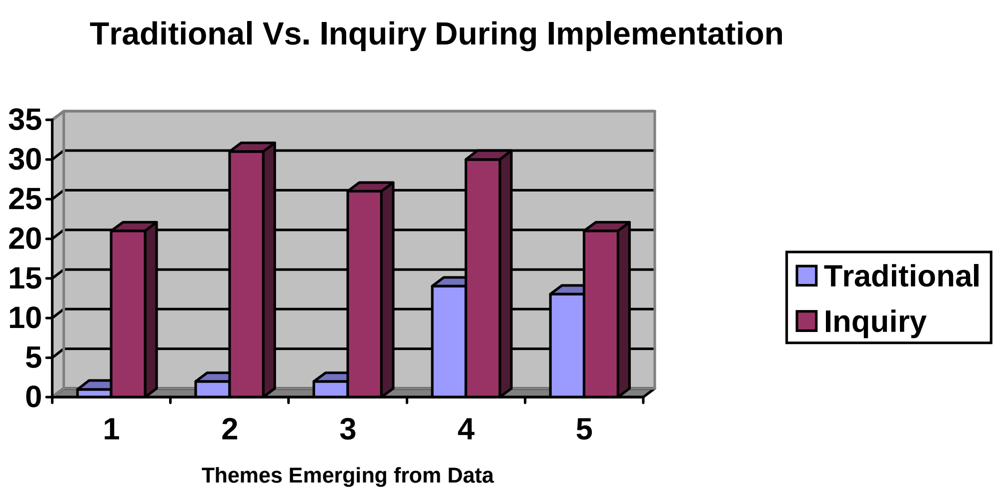
Themes Emerging From Qualitative Data						
	Use of Manipu-latives	**Math as a Discipline**	**Student Tasks**	**Teacher Role**	**Student Role**	**Teacher Misconcep-tions**
During Math Out of the Box	What we are going to do now is collect some more data. Instead of measuring around your wrist, you are going to measure around your ankle. 3.2 C If you have 28 now and you push the equals button three more times, what will your answer be? Make a prediction. 3.3B Take the pattern you are assigned and make a representation of the pattern on your board. 3.4B	Look at your numbers as you are doing it. See what kinds of relationships you can see from your numbers. 2.4B How can we make sounds to create this pattern? 3.4B Pause; predict the next number by looking at the pattern of the numbers. Don't add in your head; use the other numbers to make a prediction. 4.1B You are going to tell me any kind of growing pattern that we see during the day and we will add it to our chart. 4.4B	I want you to extend your pattern so that it goes on and I want you to have a yellow in your pattern. I want you to work with your partner. 2.2B You are going to have thirty tiles. You and your partner are going to make a pattern. 2.3 B Today we are going to analyze number sequences by playing two different games. 4.1B	We have got four words to write down. We are going to find out what each one of these words mean. 3.2C You are trying to find the strips that match the pattern you have on your magnetic board. Look and see which one of the strips matches your pattern. 3.4B What kind of suggestions could you give on how to analyze the sequence?	T: How many people are in the core? S: Six. T: No, not six. S: Three. 2.2B S: Is this right? T: Ask your partner if it is right. 2.4C Students help each other to make sure they are measuring their ankles correctly. 3.2C Students continue to discuss the numbers in your groups. 4.1B	S: The core separates a pattern from another pattern. T: Very good, it separates the two patterns. 2.1B T: The pattern is red, red, blue. This is called the core. One of these colors is the element. Red, red, blue is the pattern; there are one, two, three elements. Three elements that make up the pattern. 2.3B

				4.1B		

The statements in five out of the six themes were then recoded to identify whether the statement typified traditional or inquiry instruction. The sixth theme emerging from the data, "Teacher Misconceptions" represented gaps in content knowledge that were displayed by teachers in this setting. The qualitative data placed under this category could not be labeled as traditional or inquiry, therefore, this category was omitted from the coding process. The data in the five remaining categories were quantized to identify whether teachers in this setting were instructing in an inquiry manner[25]. The Five Dimensions of Instructional Practice instrument was utilized during this process to identify if statements were representative of traditional or inquiry instruction. Figure B shows the results from this analysis.

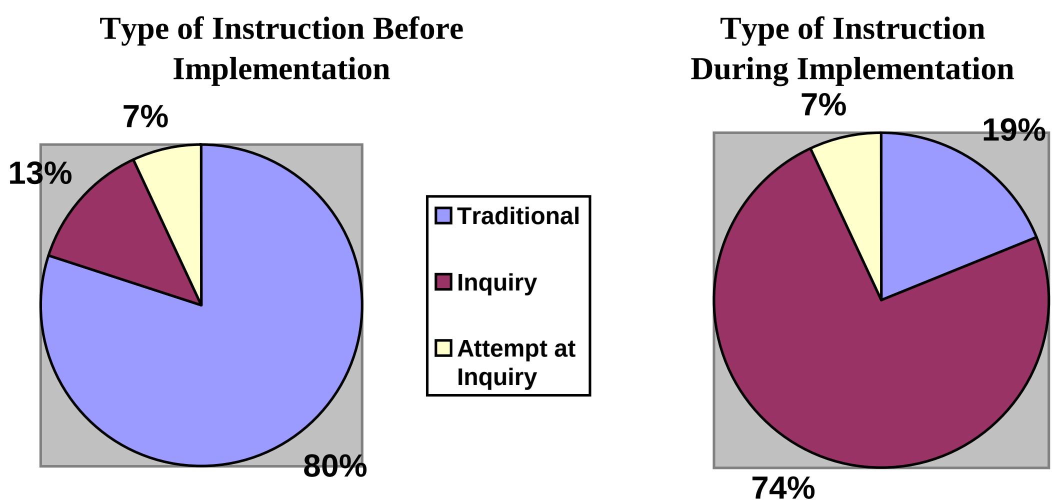
Figure B





A separate theme emerged from this second part of the data analysis. In many observations, teachers were identified as attempting inquiry instruction, but were unknowingly using traditional methods. This phenomenon could potentially be explained in two ways: the teachers in this study saw the value of both traditional and inquiry instruction and intentionally applied both in their lesson or teachers who felt that they were instructing in an inquiry manner did not have a clear picture of what inquiry should look like in the classroom. Based on the fact that teachers volunteered to participate in a study that was primarily geared toward evaluating how an inquiry based program affected student achievement, it seems that the latter explanation would apply here. Figure C represents the overall types of instruction that were observed prior to and during implementation. Instances from each category are combined to show an overall picture of the type of instruction that teachers in this setting demonstrated.

Figure C



Observations conducted prior to implementation indicate that eighty percent of the coded data show participants instructing in a traditional manner. This number would have been higher, but by recoding the data as either traditional or inquiry, researchers identified instances where teachers attempted to instruct in an inquiry manner, but were instead using traditional methods. This phenomenon occurred seven percent of the time in observations prior to implementation. Only thirteen percent of the coded data represented what could be considered inquiry-based instruction. This percentage changed to seventy-four when teachers were instructing with the Math Out of the Box program.

Based on these results, it is evident that teachers in this setting typically taught in a traditional manner prior to implementing the Math Out of the Box program. Observations taken while teachers were instructing with Math Out of the Box indicate that all teachers showed evidence of inquiry-based instruction on various levels and the majority of teachers in this study showed a complete change in instruction from traditional to inquiry during the implementation. This conclusion was strengthened from the statements teachers made about their instruction on the questionnaires and in the focus group interview.

The change in instruction that took place among these particular teachers was not made because of the new program alone. Other factors were necessary to ensure that the teachers in this study were able to change their instructional practice. Data from the interviews and questionnaires were also analyzed for themes relating to the types of support that the teachers in this setting needed to implement this program in the most effective manner. Three categories emerged: collaborative planning and reflection, a curriculum that supports inquiry instruction, and an inquiry-based professional development model. These categories were identified as methods of support for traditional teachers who wish to instruct in an inquiry manner.

Teachers in this setting found that the Math Out of the Box program provided a strong basis for teachers who are unsure of how to instruct in an inquiry-manner. Teachers often cited the amount of manipulatives and types of activities included in the program as a rationale for why their method of instruction changed. In addition to a new curriculum that supports and fosters inquiry-based instruction, it was necessary for the teachers in this study to attend three separate sessions of professional development to both gain insight on how to use the new program and provide an outlet for reflection during implementation. It was not uncommon to read teachers commenting on the type of learning that took place during professional development on their questionnaires. “I was excited about the depth of understanding that I gained in mathematics from the professional development and implementation of the Math Out of the Box program. I became more confident at teaching a variety of mathematical concepts such as algebra and data.” Many of the teachers in this study demonstrated a lack of content knowledge when instructing students on concepts in Algebra and Data. These misconceptions were common enough to warrant a separate category when analyzing and identifying themes among the data. For example, one second-grade teacher displayed a misconception that she held regarding patterns when introducing the vocabulary word “core”. During this lesson, she used the terms “core” and “pattern” interchangeably indicating that she did not have a strong grasp of the vocabulary associated with this content strand. While the professional development addressed content issues in addition to instructional strategies, it was clear that teachers required further training dealing specifically with content knowledge.

Reflection became a key piece in the process of changing instruction. Teachers cited common planning with their grade level as a way of dealing with the problems that arose when instructing with this new program. The fourth grade teachers in this setting were particularly successful in this study. One possible rationale for this easy transition was that the fourth grade team met every week and spent time planning and reflecting while using this program. It was also important that teachers had a person to go to with questions about the new program they were beginning to teach. Teachers always had the availability of email to pose questions that they had about the program. This process enabled teachers to feel more comfortable with both the new curriculum program and the practice of being observed. One final factor that seemed to aid in the change of instructional practice from traditional to inquiry was the willingness to do so by the participants in this study. By volunteering for this study, the teachers that participated were actively involved in changing their instructional practice.

possibilities for future investigations:

Future studies on changes in instructional practice with specificity to the Math Out of the Box program could look deeper at the practices involved in planning for instruction both alone and with grade level teams. It would be beneficial to develop a planning model for teachers to use when implementing inquiry-oriented mathematics curriculums. Other possible studies include researching the effects of adding a mentor as a resource for teachers while implementing new curriculum programs or analyzing the community in a classroom using an inquiry-based instructional program. It would also be beneficial to continue studying the teachers at this particular sight to assess if the change in instructional practice continues over an extended period of time. One important insight that developed as a result of this study was the amount of mathematics content misconceptions teachers in this setting displayed. Lack of content knowledge among elementary teachers has been a common theme in the literature relating to educational research[26,27,28,29]. In fact, many of the federal programs designated to improve the quality of STEM education are related to increasing teacher content knowledge at the elementary and middle school level[2]. This lack of knowledge was even more apparent as teachers in this setting made the transition from traditional to inquiry. In fact, teachers themselves became cognizant of their lack of knowledge as demonstrated through the types of discussion that were occurring during common planning time. The fourth grade teachers in this setting who planned together on a weekly basis began to discuss content related issues during planning sessions rather than focusing specifically on outlining tasks. Research conducted on how these teachers react to professional development geared primarily toward content misconceptions would be beneficial for future teachers implementing this program. A final possibility for future investigation is to examine if inquiry-based instruction supports student learning in all areas of mathematics. These types of studies have been done on a small scale, but a longitudinal study examining the impact of inquiry-based programs on student achievement is necessary to ensure the validity of this type of instruction.

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Appendix A

Math Out of the Box
Developing Algebraic Thinking
Reflection Day

School:
ID Code
2 digit birth month, 2 digit birth day, 1 digit grade level ____ ____ ____ ____ ____

1. **Write about something mathematical that you learned during this Math Out of the Box project- in the professional development sessions, when working with other teachers in your school, or while planning and teaching the lessons for your students.**
2. **Describe how or if your teaching changed during this project.**
3. **Describe something new that you learned about your students while teaching Math Out of the Box.**
4. **Describe the most challenging and/or the most rewarding aspects of participating in this project.**
5. **Complete this chart:**

Before using MOOTB	While using MOOTB
Describe a typical math lesson in your classroom	
Describe who typically does the "mathematical talk" in your classroom.	
Describe how manipulatives, technology, and other materials are used in your classroom.	

Appendix B

Five Dimensions of Instructional Practice	
Traditional ◄—— **Dimensions of Practice**	——► **Inquiry-based**
D1: Teacher's conceptions of math as a discipline	
a. Emphasis on procedures and "basic facts" in teaching. b. A belief that skill mastery implies mathematical understanding. c. Using or teaching a single strategy for solving certain types of problems. d. Seeing mathematics as a checklist of topics or procedures to learn.	a. Emphasis on conceptual development in teaching. b. Explaining ideas using a variety of representations implies mathematical understanding. c. Recognizing connections that lead to the use of multiple strategies to solve problems. d. Seeing mathematics as a coherent body of knowledge, relating to everything—as a means to understanding and describing the world.
D2: Students' tasks	
a. Problems have a single solution and a single path to the solution is presented. b. A problem may have multiple paths for reaching a solution, but the teacher demonstrates and encourages the use of a particular path over all others. c. Lesson tasks tend to provide practice at replicating a single method. d. Students are expected to use particular paths to solve particular problems and are given little autonomy exploring new paths for solving problems.	a. Problems and tasks are designed to have multiple solutions and/or multiple paths for getting to a single solution. b. Students are encouraged to share and compare strategies for solving problems. c. Lesson tasks involve both reasoning and proof in the development of mathematical ideas. d. Students have autonomy within defined parameters to discover, explore, and create multiple paths to solutions.
D3: Teacher's role in curriculum implementation	
a. The teacher practices a model of transmitting content to the students through instructional strategies of "telling" and "demonstration." b. The teacher is considered the expert and she "explains" and directs how things should be done. c. The content of the lesson is focused on teacher defined and demonstrated procedures that students are expected to replicate. d. The teacher's questions focus on students' ability to replicate previously defined procedures and facts, and to provide correct final answers. e. The process standards may be included in the lesson, but the process itself is not used for the construction of knowledge—it is just another way of performing a known procedure. f. The teacher evaluates the students' work and answers for accuracy.	a. The teacher uses "indirect" strategies for instruction, allowing students to explore and investigate ideas within defined parameters and provides clarification and summarization of ideas as appropriate. b. The teacher acts in several different roles including facilitator, questioner, and co-learner. c. The content of the lesson is focused on development of a mathematical idea that students explore and investigate through a variety of means. d. The teacher's questions focus on student thinking in the development of new ideas and on probing for the logic and reasoning that lie behind the answers. e. The process standards are used as a means for investigating and exploring mathematical ideas in order to construct knowledge. f. The teacher assesses students' work and students' comments for understanding.
D4: Student interactions and confidence	
a. Most interactions that occur in the classroom are teacher to student or student to teacher. b. Students do not recognize peers as sources of mathematical information. c. The teacher views group work primarily as an opportunity to develop social skills or as a necessity for sharing materials. d. Students rely upon the teacher for judgment of quality and accuracy of work.	a. Interactions are a blend of student-to-student, student to teacher, and teacher to student. b. Students recognize themselves and their peers as sources of mathematical information. c. The teacher values the culture of group work as an environment for learning. d. Students and teachers have confidence in students' ability to assess the quality and accuracy of and their peers' work.
D5: Use of manipulatives and other tools in classroom	

a. Materials are used by the teacher for demonstrating a specific procedure to be used in generating answers. b. The teacher directs and prescribes the use of materials and students replicate the actions of the teacher. c. Materials are viewed by the teacher as the “concrete” step that leads to “pencil and paper” abstraction—as a concrete “crutch” needed by “low level” students who cannot do the “abstract” pencil and paper work.	a. When materials are used by teachers for demonstration purposes, it is either to define parameters of proper use or to explore an idea collectively as a whole class. b. Students have access to materials in order to explore and investigate mathematical solution paths, relationships, and connections. c. Materials are viewed by the teacher as “thinker toys” used by students to explore new ideas, or as “assessment tools” used by students to demonstrate constructed knowledge.